

The Effect of Problem Posing Instruction on 9th Grade Students' Mathematics Academic Achievement and Retention

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ABSTRACT The purpose of this research is to examine the effects of problem posing intervention on 9th grade students' mathematics achievement and retention. The subject triangles were used in the research as a tool to observe the differences between experimental and control groups. Effects of problem posing instruction analyzed by specially designed tests on pre and post activities. Meanwhile, sought student responses were through individual meetings. This study has been conducted with 9th grade students at a Kazakh High School for gifted students during the second semester of 2012-2013 academic year. 60 students were divided into two groups. One of the groups was experimental and the other was control group. There was equal number of students in each group, that is, 30. The research duration was two months in the same school. Within this research, a mixed methods design is used with quantitative and qualitative components. Data from quantitative component that was pre and post test were analyzed by using SPSS computer package. Qualitative design included data through which students were compared from pre to post intervention opinions. The Mathematics Achievement Test were used in order to measure the students' mathematics academic achievement and retention as well. The reliability of the tests was measured by special techniques and the value of Cronbach's Alpha constant was calculated as 0.93 for achievement test. During the problem posing instruction with experimental group students we used the activities that were specially designed triangles problems in the light of problem posing stages. Traditional educational methods were used in the control group. In addition, some questions were prepared for the students who got extreme scores from the activities. At the end of the research, data was evaluated by using paired sample t-test and the analyses of interview with students were conducted by using the descriptive methods.

INTRODUCTION

Educators and researchers are trying to find new methods in teaching and learning of mathematics education to improve and develop the students' problem solving abilities. Today many educators around the world agree that traditional methods of teaching and learning process cannot prepare individuals to the future. The aim of teaching mathematics is to develop cognitive abilities of children, logical thinking, self sufficiency and empowering the memory. Meanwhile to develop creative activities; the ability to observe, compare, find similarities and differences; the ability to analyze, synthesize, generalize, abstract; the skills of mental arithmetic; the skills of proper and logical mathematical language.

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Generally, all curricula about teaching and learning mathematics are agreed that the aim of teaching mathematics is to extend the students' ways of learning and to develop the students' abilities in problem solving and provide applicable mathematical knowledge, expertise and skills for future needs. Particularly problem solving is accepted as the heart of mathematics education (NCTM 2000). The students should understand their environment and world together and they should apply what they learn to real life. They have to use mathematical skills and mathematical knowledge in modern society. Otherwise students with traditional methods cannot solve the problems and cannot make relations between real life and their learning in rapidly changing world (MEB 2011). Instead of teacher oriented methods, student oriented methods should be discussed. One of them is problem posing approach in math education.

Problem posing is not independent from problem solving (Cai and Hwang 2002). There is a strong relationship between problem solving and problem posing as a cognitive process

(Lowrie 2002). Brown and Walter (1993) suggested a new approach to problem posing and problem solving in mathematics teaching by using the "What If Not" (WIN) strategy. The strategy is based on the idea that modifying the attributes of a given problem could yield new and original problems that may give very interesting results. In this approach, the students are encouraged to go through three levels starting with the examining the problem to generate new problems. At the first level, the students are asked to write the list of the problem attributes. And at the second level the students should ask many questions about the attributes related "What If Not" question and then suggest alternatives to the listed attributes. The last level of problem posing, they pose new questions by making more generalization.

Brown and Walter (1995) also stated that one of the important consequences of mathematics education is to provide opportunities to the students in mathematics lessons for developing their problem posing skills. Because problem posing is not only to generate new problems from given situations but also reformulate given problem and generalize for the solution. Problem posing has too much interest because of its effect in creativity and mathematical ability (Silver 1994). Problem posing in contrast to traditional problem solving methods reduces anxiety and common fears about mathematics and increases positive attitudes toward mathematics (Nicolau and Philippou 2004). Problem posing improves not only students but also teachers' attitudes; alleviate misunderstanding about the nature of mathematics. Problem posing activities give more responsibility to the students who are motivated for the problems during the mathematics class. Problem posing methods of learning bring up the students for the future as social an individual that meets the expectation of modern society.

Problem posing activities not only enrich the students' mathematical abilities but also develop positive attitudes toward mathematics that yields mathematics achievement. There are many researchers who reported that high correlations are found between problem posing and mathematics achievement (English 1997; Leung and Silver 1997; Silver and Cai 1996). According to Pollak (1987), students sometimes encounter ill structured problems that have been restructured during problem posing activities give more understanding to the students. It is

interesting that ill problems increases mathematics achievement. Because students reconstruct the ill questions by which they discover the new ways to solve the questions. The approach of Freire (1972, 1973) about problem posing is also another factor of mathematical achievement is that problem posing education increases the motivation of the students to participate in the lessons because of freedom in educational classes, that's why students would have high level of thinking about their environment. The main element in problem posing is critical thinking that gives the students more chance to recommend open ended and non-routine questions. Understanding the mathematical concepts and mathematical process during problem posing stages are also positive factors in mathematics achievement.

Mathematical achievement is also influenced by students' thinking, problem solving skills, attitudes and confidence in mathematics (English 1997). Another factor in mathematics achievement is the students' interest toward mathematics, while the students' interest increases toward the activities, tasks become easier for students. Silver (1994) stated that problem posing is an alternative method for the students who are not good at mathematics. Mathematics achievement is just knowledge and some skills that are taught by teachers during the lessons in a period of time (Akay 2006). To measure the mathematics achievement educators designed some achievement tests that are standard achievement tests and teacher made tests (Mehrens and Lehmann 1987). While problems are posed by students, they realize the main and important structure of the problems and discover the clues in the process of solution. Dickerson (1999) claimed that problem posing methods increased the mathematics achievement of the students by listing attributes of the cases that is from real life situations and from reasonable tasks.

MATERIAL AND METHODS

Research Design of the Study

In this part, research model, participants, measurement instruments, types of application of research design, data gathering and evaluation of collected data are considered. In this study, quantitative and qualitative methods were used. In quantitative research, theory-based instruc-

tions and traditional methods of instruction were the independent variables, while the Achievement Test was the dependent variable. In this study, the effect of independent variables on dependents variable will be controlled. That's why the present research was mostly an experimental study. The study used the matching-only pre tests-post tests control group design. After 40 days we applied the Mathematics Achievement test as Retention test for both groups in order to make the effect of theory based instruction on retention for both groups.

In Table 1, the abbreviations have the following meanings:

- EG represent s experimental group that received instruction with the "Theory-based Instruction" (TBI).
- CG represents the control group, which received instruction with the "Traditional Method" (TM).

Table 1: Research design of the present study

<i>Group</i>	<i>Pre-test</i>	<i>Treat</i>	<i>Post-test</i>
EG	M1	TBI	M1
CG	M1	TM	M1

The measuring instruments in are the following (Table 2):

- M1: Mathematics achievement test (MAT);
- MAT was administered as pre- and post-tests.

Table 2: Distribution of the students

	<i>Experimental group</i>	<i>Control group</i>
Number of students	30	30

In experimental group theory based instruction was used while traditional teaching methods were used in control groups. In both groups, before and after instruction, the Mathematics Achievement Test was used as pre- and post-test. Before instruction, all students took Mathematics Equivalent Test. This was used in order to make the groups equivalent average. This means that experimental and control group service teachers were formed such that both groups had roughly equal average achievement levels. In the qualitative part of the study, descriptive model was used to figure out how TBI instruction affected the mathematics achievement, in

the same part, interviews were done with the students which were recorded by the researchers.

Participants

The students were from Aksai Kazakh high school for gifted children. The total number of students participating in the study was 60 from 9th grade. We divided the students into two groups: "experimental" and "control" according to Mathematics Equivalent Test results as described above. All of the subjects learned the same mathematical content that consisted of triangle problems based on the same textbook in the same period of time. The study was carried out during the spring quarter of 2012-2013 academic years. In the qualitative part of the research, some students were selected for interview conversations. All experimental group students stated their opinions about the theory based instruction. Their opinions on problem posing materials were recorded on video. The order of interview was determined based on average quiz scores received during the problem posing instruction.

Definition of Terms

In this section, some of the terms that were used in this study are defined to make the study report more clear:

1. Mathematics Achievement refers to subject's achievement score on triangle problems that was measured by MAT, which was applied before and after the instruction.
2. Problem posing instruction refers to formulation of a given case, creation of a new theory or problem and prove it in some ways.
3. Treatment refers to the method of instruction; either the instruction is given by Theory based (TBI) or by instruction with Traditional methods.
4. Control Group (CG) refers to the group which received instruction using the Traditional Method.
5. Experimental Group (EG) refers to the group which received instruction on problem posing based.
6. Mathematics Equivalent Test refers to subject's achievement scores on a general review of first grade mathematics topics before study.

7. Retention test refers to mathematics achievement test that was applied to the same groups after 40 days

Steps of the Study

1. Before we began the study, we applied a Mathematics Equivalent Test to divide the groups according to the adjustment level. Their average results were compared and according to the results, participants were divided into two groups: the experimental and control.
2. The Mathematics Achievement Test (MAT) was given to the 9th grade gifted students. The mathematics achievement test that included triangle problems was prepared by an experienced teacher and was controlled by experts. The test included 30 questions that were grouped according to the topics: triangular basic concepts, types of triangles related with properties of triangular properties. The problems include the concept on the theorems discussed before. The questions were mostly selected from books that are suitable for the annual program. The difficulty level of the test was assessed after the results were available.
3. The MAT was piloted with 128 students from 10th grade high school students in the same school in Almaty in 2012. This pilot study provided a test of reliability and validity of MAT. According to the results of this pilot study, the MAT was revised.
4. Activity sheets were prepared using appropriate problem statements as recommended by reports of research found in the literature.
5. Mathematics teachers administered the MAT to the students before and after the treatment during a mathematics lesson. The MAT test was applied to both groups before and after the study.
6. The study was run for a period of seven weeks with 8 hours per week
7. The data obtained from the MAT administered before and after the study was recorded at the end of the study.

Data Collecting Instruments

Data was collected in this research using the following instruments:

1. Mathematics Achievement Test: The test included the questions about triangle problems that were grouped under the subjects of properties of constructing triangles related with geometrical approach itself. The questions were prepared by the researchers and were controlled by the experienced teachers and the adviser.
2. The reports of students in the experimental group: These were about the problem posing instruction based instruction method as an application of the method on triangular problems.
3. The video recording of the students in the experimental group.

Mathematics Achievement Test

The test was prepared according to the curriculum stated by Ministry of Education of Kazakhstan for mathematics department. There were 30 problems in the test. The problems covered the all subtopics of triangular problems in the syllabus. The concepts found in triangular problems had mathematical context. The reliability and validity measurement was done and the level of the problems was controlled by experts. To evaluate the reliability and validity of the Mathematics Achievement Test, it was first applied on 128 high school students in the same school. Factor analysis of the test and the problems was evaluated by SPSS computer program. After the test was completed, Mathematics Achievement Test was applied to the students as a pre-test before the instruction and as a post-test after the instruction with problem posing and as a permanent test after 40 days. The test was prepared in two variants to prevent the math students from copying solutions from each other. Enough time was provided to the students to solve the problems. As it was mentioned above the test was applied on 128 high school students from SDU. The difficulty level and differentiable properties of the problems were analyzed by standard calculations. Three problems were taken out from test as they were very low at difficulty index. The analysis was as follows: 27% of the papers were selected from the top of the results list and another 27% of the papers were selected from the bottom. For each problem the coefficient of difficulty index p and the coefficient of differentiable index r were calculated using the following equations:

$$p = r = \frac{\text{ncf} - \text{ncl}}{\text{nop}}$$

p- Coefficient of difficulty index;
 r- Coefficient of differentiable index;
 ncf - Number of correct answers from first 27%;
 ncl- number of correct answers from last 27%;
 nop- total number of answer sheets.

After evaluating the coefficients r and p problems were changed and the test was applied again to a different group of students. Thus the last and reformed test for research was prepared consisting of 30 problems. There were 34 students from first and last groups from the results list, 30 problems, 128 participants. The average value was 21.3 and the standard deviation was 6.7. The reliability of the test was also measured by calculating the Cronbach's Alpha constant. We found that Cronbach's Alpha $\alpha=0.930$ for our test. Based on this value, this we can claim that Mathematics Achievement test is suitable for our research. We found the alpha by using the SPSS program: Correct (1) and wrong (0) answers of the students.

RESULTS

In this part the results of mathematics achievement test and the effect of retention will be discussed. Meanwhile the writings and video presentations of the students about problem posing will be evaluated. Recommendations of the students about problem posing in individual meetings will also be presented. The averages and standard deviations of pre and post test shown in the table for experimental and control groups.

In order to analyze the effect of problem posing instruction on students' academic

achievement, the findings acquired in pre and post application of the academic achievement test to the research and control groups were drawn in tables, and some comments were made in parallel to these findings. The arithmetic mean of the pre-test scores taken by the experimental group students was found 12.33 and the respected figure of the control group students was found 9.66 (Table 3). It is observed that there is a less point difference between group means and p value is more than 0.05. This indicates that there is no significant difference at the 0.05 confidence interval between the pre-test scores of the research group and control group students. By the fact that there is no significant difference between the pre-test scores of the experimental and control group students, the condition concerning the nearness of pre-knowledge level of the experimental and control groups before the research is fulfilled.

When post-test scores of the experimental group and control group students were examined (Table 4), it was found that the arithmetic mean of the post-test scores taken by the experimental group students was 17.18 and the respected figure of the control group students was 11.03. It can be seen that there is more point difference between group means and p value (0.007) is less than 0.05. This indicates that there is a significant difference at the 0.05 confidence interval between the post-test scores of the research group and control group students on behalf of the former group.

Findings about Retention

In this part the researchers will try to find relations between Mathematics Achievement

Table 3: The sample paired t test results of experimental and control group students for mathematics achievement pre test

<i>Pre-test results</i>	<i>N</i>	<i>X</i>	<i>SD</i>	<i>SE</i>	<i>df</i>	<i>t</i>	<i>p</i>
Experimental G	30	12.33	3.74	0.78	25	0.17	0.861
Control G	30	9.66	3.01	0.67			

Table 4: The sample paired t test results of experimental and control group students for mathematics achievement post test

<i>Pre-test results</i>	<i>N</i>	<i>X</i>	<i>SD</i>	<i>SE</i>	<i>df</i>	<i>t</i>	<i>p</i>
Experimental G	30	17.18	5.49	0.67	26	2.92	0.007
Control G	30	11.03	3.59	1.06			

Test and retention test for both groups. This data was obtained from mathematics post achievement test and the retention test that was applied 40 days later to both experimental and control group students. The results of retention test for control group (X retention) = 10.11 while the result of post-test was 11.03. There is a little difference between the two results (Table 5). The retention was mostly affected by the activities done in the class. Results showed that students who took traditional instruction showed less differences between post test and retention test.

Table 5: The mathematics achievement post-test and the retention test results of control groups

Post-test results	N	X	SD	SE
Retention	30	10.01	2.95	0.56
Control (post)	30	11.03	5.5	1.09

The results indicate that the average of retention test has a higher mean score for experimental groups ($X=14.03>10.11$) and $p=0.02<0.05$ so we have a significant value (Table 6). Thus we can claim that problem posing has more positive effect on retention than any kind of traditional methods.

Table 6: The mathematics achievement post-test and the retention test results of the experiment groups

Post-test results	N	X	SD	SE
Retention	30	14.03	2.29	0.44
Experiment (post)	30	17.18	3.49	1.09

From the data presented it can be concluded that the problem posing instruction has more positive effect on retention of triangular problems than classical teaching methods. Retention of mathematical subjects is mostly affected by the teaching methods. Here, the statistical differences indicate that the problem posing techniques used during the process of teaching is a tool of considerable effect. Some of the opinions expressed by students from this group during the activities carried out in the research process are given below.

Student 1: I liked problem posing class very much. The posing activities were enjoyable and more interesting. The problem posing activities made us to like problems. Group works were also good as well. I was very contented with the applications.”

Student 2: “We were motivated more to study by writing the questions of problem posing that were fun and learning. In the problem posing method, it is so easy to produce questions and it is so good and fruitful to add our own ideas and discuss within group.”

Student 3: “The problem posing instruction attracted me to the participation to the class activities. It is a good method. We both learn and have fun. We started to like solving problems.

Student 4: “I overcome the fears and anxieties about mathematics problems in problem posing lessons. I understand subject matters better. The problems seem to be so easy.”

It can be said that there is no positive improvement in the control groups to which traditional teaching methods were applied. Yet, problem posing type of education employed in the experimental group brought about positive improvements in the conceptual development of the students. In the experimental group in which problem posing activities are applied, since students are in communication with their group members and other groups, they could find the opportunity to discuss and share their ideas. In this way, information transfer among students is accomplished. The examples given are chosen out of daily life and they are enriched by students.

DISCUSSION

Problem posing method of instruction has significantly increased students’ mathematical academic achievement and have significant visual effect on retention. The students who have been experimental class had high positive attitudes toward mathematics. In problem posing instruction, students were not motivated not only finding the correct answers of the problems but also the ways that they followed through the solution period of the questions. They were also more social when they tried to pose the problems. This was provided by interaction with the students as well as with teachers. The students had a chance to ask questions to teachers that is why they cancelled some misconceptions and they were directed right way during the problem posing stages.

The result of this present research indicated that contrary to traditional teaching methods, problem posing instruction produces significant-

ly positive results in students' attitudes toward word problems and mathematics and mathematics achievement. Active involvement and more participation gave the students more confidence and positive attitudes. Eggen and Kauchak (2003) emphasize that problem posing instruction emphasizes students' active involvement in learning. Thus, problem posing teaching frequently express that the students learn by connecting new knowledge to the real world. Our findings were similar with English (1997) claimed that the activities of problem posing had a strong emphasis on children being creative, divergent, and flexible in their thinking and students were encouraged to look beyond the basic meanings of mathematics with those activities. It can be claimed that there is a strong relation between interest and effort that is increase in motivation because problem posing instruction is based mostly on students who start to be interested people. Interest and motivation by this way can be formed together; interest produces motivation and motivation produces interest. Problem posing increases motivation and optimism (Brown and Walter 1983). Therefore, we can say that problem posing has a positive influence on self efficacy. Moreover problem posing reduces anxiety that is a negative factor on self efficacy beliefs. Problem posing which gives students more freedom and dialogue with the teachers provides a good development for self confidence. Kliman and Richard (1992) accepted that problem posing enlarges the inner control of the students. Inner control is an effective component of self attitude construction.

CONCLUSION

In conclusion, problem posing instruction proposed new teaching methods in order to teach word problems in mathematics education. The results of the study also showed that traditional teaching methods cannot give them to the students. Because traditional methods do not cover the attitudes of the students that were basically can't consider the psychological sides of the students. It may be just concentrated the mathematics achievement. Apparently, in both type of educational system the role of the teacher can't be neglected. In addition to all parts of problem posing, we should not forget that the main aim is not to create the best problem posers.

Instead of this we need to use problem posing as a tool to produce good problem solvers.

RECOMMENDATIONS

Following are some recommendations for further research on the effects of theoretical instruction on students' mathematics achievement in mathematics education.

The same kind of research may be done with the candidate teachers who are taught mathematics in their own languages. In this research, the students learned mathematics in English. Teaching in mother language may have some effects on student achievement. On the other hand, the number of candidate teachers participating to our study was 60. The number of students may be increased in order to get more statistically significant results.

1. In our present research was conducted with local students. Further studies may be conducted with foreign academic students to generate new conclusions.
2. The study may be done with different grades and different high schools, as well as with some primary classes .
3. The study took 8 weeks to collect the data and to finish the problem posing instruction materials. Longer studies can produce new insights.
4. Kazakhstan is suitable to do new researches with the students of different cultural backgrounds. If the number of participants from diverse cultures were increased, our research would be more realistic and satisfactory.
5. New research may be done with different subjects not only triangular materials, designed with perspectives of other students.
6. The study was especially important for pre-service teachers that played essential role in students' learning process. So that's why we should conduct additional research for different grades
7. Another mathematical subject may be chosen as the research subject. The same relations can probably be shown in the case of mathematical achievement, mathematics attitudes and self-efficacy beliefs.
8. Another variable that may be researched with problem posing based teaching is retention and how results differ at old ages.

9. The effect of the research might be studied on gifted students because there are not enough studies of theory based on gifted students. This can be very useful to prepare an annual plan for special students. It can also be interesting for the educators who are interested in gifted students.

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