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Conservativeness in Tobacco Smoke Spread Process

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ABSTRACT After a cigarette has been smoked in a limited area and that the smoke has dissipated, people, especially passive smokers sometime have the visual illusion that smoke particles and particulate matters have gone and that the danger to the health and lungs has ended. In this paper, a theoretical approach is used to investigate the quantitative and qualitative effects of smoke particulate matters, transported and spreading in a limited region. The researcher proves mathematically that tobacco smoke and its toxic chemicals remain in a house's room or public place where they have been released. Transport and break-up processes of atmospheric particulate maters do not removed them but only change the distribution of the particulates in the room's atmosphere, hereby addressing the problem of people exposed to second-hand tobacco smoke in our homes and work places.

INTRODUCTION

There are more than 1.1 billion people smoking cigarettes around the world and this number continues to rise. Tobacco use remains the first global cause of preventable death. It kills almost 6 million people every year and causes a considerable damage to the economy of the concerned nations worldwide. Most of these deaths occur in developing or emerging countries, and is expected to gradually increase over the next coming decades.

Cigarette smoke is the major cause of lung cancer and it is well known that it can also be responsible for the spread of it over the whole body (World Health Organization 2009, 2011; Surgeon General's Report 2014). Exposure to smoke, irrespective of whether you are active or passive smokers is catastrophic for the health. Second-hand tobacco smoke is proven to be more toxic for passive smokers, especially for young kids and teenagers. Indeed, adults are not the only people who suffer because of smoking. Today, about half of all the children between ages 3 and 18 years in many countries are exposed to cigarette smoke regularly, either at home or in places such as restaurants that still allow smoking (Surgeon General's Report 2014). Second-hand tobacco smoke is defined as the smoke emitted from the burning end of a cigarette or from other tobacco products, usually in combination with the mainstream smoke exhaled by the smoker, and has similar components to inhaled or mainstream smoke (World Health Organization 2009, 2011). It is then seen as a combination of both sidestream and mainstream smoke emissions. However, it is three to four times more toxic per gram of particulate matter than mainstream tobacco smoke, and the toxicity of sidestream smoke is higher than the sum of the toxicities of its constituents. Emissions from second-hand tobacco smoke contain more than 50 carcinogenic chemicals resulting from thousands of billions of airborne solid and liquid particulates and gases transported and spreading during the smoking process. The mode of particle size distribution during tobacco smoking process is generally the accumulation of particulates ranging between 75–250 nm and formed by the processes of coagulation and fragmentation, completed by the process of transport (spreading) (Amholland 1983; Mark 2006). During the transport, the smoke particulates may coagulate or partially split and deposit on surfaces and walls through diffusion and sedimentation.

In mathematical modeling theory, this means we are in presence of coagulation dynamics combined with fission and transport processes. Mathematically, fission and transport processes are expressed by the integrodifferential equation (Doungmo Goufo et al. 2013; Okubo et al. 2001)

$$\frac{\partial}{\partial t} u(t, x, m) = -a(x,m)u(t, x, m) + \int_m^\infty b(x, s, m)\, a(x, s)\, u(t, x, s)\, ds - div\, (w(x,m)u(t, x, m)) \quad (1)$$

where in terms of the mass size m and the position x, the state of the system is characterized at any moment t by the particulate-mass-position distribution $u = u(t, x, m)$, (u is also called the *density* or *concentration* of m- particulate matter at position x at time t), with

$u: \mathbb{R}_1 \times \mathbb{R}^3 \times \mathbb{R}_. \to \mathbb{R}$ The three dimensional vector $w = w(x, m)$ represents the velocity of the transport and is supposed to be a known quantity depending on m and x; The rates a and b are defined as in equation (2) below. Mathematical systems modelling fragmentation combined with transport have been intensively used to describe a wide range of phenomena appearing in chemical engineering, ecology or aquaculture and polymer sciences. Concrete applications (see (Doungmo et al. 2013; Oukouomi 2013; Doungmo 2013; Edwards et al. 1990; Huang et al. 1991) and references therein) include solid drugs break-up in organisms or in solutions, rock fractures, breakage of particulates and external processes such as oxidation, melting or dissolution.

In this paper, the researcher focuses on the transport process in the spread of cigarette smoke and the related break-up of bigger particulates sizes to form smaller ones in order to quickly spread in a room or public place.

METHODOLOGY

Conservativeness During the Break-Up of Tobacco Particulates Matter

A specific smoke particulate of size n (also called n-particulate) can disappear due to its fission or appear due to the scission of a m-particulate, with $m > n$. Mathematically, the evolution of the number density of smoke particulate matter breaking up during a smoking process can be extracted from (1) to give

$$\frac{\partial}{\partial t} u(t, x, m) = -a(x, m)u(t, x, m) + \int_m^\infty b(x, s, m)a(x,s)u(t,x,s)\, ds. \quad (2)$$

To analyze the model, the researcher sets the initial condition

$$u(0, x, m) = u^0(x, m), \text{ a.e. } (x, m) \in \mathbb{R}_3 \times \mathbb{R}_1. \quad (3)$$

where $a(x, m)$ is the average fission rate, that is, it describes the ability of a m-particulate at position χ to break into smaller particulates. Once an s-particulate at position χ breaks, the expected number of daughter particulates of size m is the non-negative measurable function $b(x, s, m)$ defined on $\mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}$. The space variable χ is supposed to vary in the whole of R^3 and the function $u^0(x, m)$ represents the density of m-particulates at position x at the beginning $(t = 0)$.

Since a particulate of size $m \leq s$ cannot split to form a s-particulate, the function $b(x, s, m)$ has its support in the set

$$\mathbb{R}^3 \times \{(s, m) \in \mathbb{R}_4 \times \mathbb{R}_4 : m < s\}. \quad (4)$$

After a s-particulate breaks up, the sum of masses of all daughter particulates should again be s, hence it follows that for any $s > 0$, $x \in \mathbb{R}^3$.

$$\int_m^s mb(x, s, m)\, dm = s. \quad (5)$$

Because the space variable χ varies in the whole of $\mathbb{R}^3$ (unbounded) and since the total mass of $\frac{a}{a\,d}\, N(t) = 0^s$ all particulates is not modified by interactions among them, the following conservation law is supposed to be satisfied:

$$N(t) = \int_R \int_0 u(t, x, m)\, mdmdx, \quad (6)$$

where is the total mass of the whole system. Since $u = u(t, x, m)$ is the density of particulate of size m at the position χ and time t and that mass is expected to be a conserved quantity, the relevant Banach space to work in is the space.

$x_1 : = L_1(\mathbb{R}^3 \times \mathbb{R}_4, mdmdx),$

But because uniqueness of solutions of (2)-(3) proved to be a more difficult problem, the researcher restricts the analysis to a smaller class of functions, by introducing the following class of Banach spaces (of distributions with finite higher moments)

$$x_1 : = L_1(\mathbb{R}^3 \times \mathbb{R}_4, m^R dmdx), r \geq 1,$$

which coincides with χ_1 for $r = 1$ and is endowed with the norm $\|.\|$ It is assumed that $u^0 \varepsilon \chi_r$ and for each $t > 0$, the function $(x, m) \to u(x, m) = u(t,x, m)$ is from the space χ_r with $r > 1$. When any subspace $s \subseteq \chi$ then, S_+ will denote the subset of S defined as. Note that any $g \varepsilon (X_r)_+$ will possess moments

$$S_+ = \{g \in S; g(x,m) \geq 0, m \in \mathbb{R}_+, x \in \mathbb{R}^3\}$$
$$M_q(t) := \int_0^\infty m^q g(t,x,m)dm$$

of all orders $q \varepsilon [O, r]$. In X_r, it is defined from the expressions on the right-hand side of (2), the operators A and B by

$$[Ag](x,m) := a(x,m)g(x,m), \quad D(A) := \{g \in \mathcal{X}_r : ag \in \mathcal{X}_r\}; \quad (7)$$

$$[Bg](x,m) := \int_m^\infty b(x,s,m)a(x,s)g(x,s)ds, \quad D(B) := D(A). \quad (8)$$

With the above settings, the following is stated:

Results

Result 1

The fission model described by (2) - (3) is formally conservative, that is, the law (6) is satisfied.

Proof: It is obvious that the operator *(A + B, D(A)* is well defined. In fact the condition (5) can be used to show that

$$s^r - \int_0^s m^r b(x,s,m)\,dm = s^r - \int_0^s m^{r-1}\, mb(x,s,m)\,dm \geq s^r - s^{r-1} \int_0^s mb(x,s,m)\,dm = 0.$$

Hence

$$\int_0^s m^r b(x,s,m)\,dm < s^r \tag{9}$$

for $r \geq 1$, $m > 0$. Note that the equality holds for $r = 1$. For every $p \,\varepsilon\, D(A)_+$, changing the order of integration by the Fubini theorem yields

$$\begin{aligned} \| Bu \|_r &= \int_{\mathbb{R}^3} \int_0^\infty [Bu](x,m) m^r\, dmdx \\ &= \int_{\mathbb{R}^3} \int_0^\infty \left(\int_m^\infty b(x,s,m) a(x,s) u(x,s) m^r\, ds \right) dmdx \\ &= \int_{\mathbb{R}^3} \int_0^\infty \left(\int_0^s b(x,s,m) a(x,s) u(x,s) m^r\, dm \right) dsdx \\ &\leq \int_{\mathbb{R}^3} \int_0^\infty a(x,s) u(x,s) s^r\, dsdx \\ &= \| Ap \|_r \\ &< \infty, \end{aligned} \tag{10}$$

where the researcher has used the inequality (9). The result follows from the fact that any arbitrary element u of $D(A)$ can be written in the form $u = u+ - u_-$,where $u+$, $u_- \,\varepsilon\, D(A)_+$. Then $\| Bu \|_r \leq \| Au \|_r$, for all $u \in D(A)$, so that *D (B): = D (A)* can be taken and *(A + B, D (A))* is well-defined. Hence, from (10) the law (6) is obtained.

$$\frac{d}{dt}\mathcal{N}(t) = \frac{d}{dt} \int_{\mathbb{R}^3} \int_0^\infty u(t,x,m) m\, dmdx = \int_{\mathbb{R}^3} \int_0^\infty m \frac{\partial}{\partial t} u(t,x,m)\, dmdx = 0$$

This proves mathematically the break-up process observed during the spread of cigarette smoke does not remove atmospheric particulate matters in the environment and that the visual disappearance of smoke is just an illusion.

Conservativeness during the Transport of Tobacco Particulates Mater

Tobacco particulate maters are largely transported from one area in a house room to another during a cigarette smoking process. This dynamic is mathematically expressed as follows: If it is defined a spatial dynamical system in which locally group-size distribution can be estimated, but in which it is also allowed immigration and emigration from adjacent areas with different distributions, the researcher obtains the general system modeling smoke particulates mater transport, extracted from (1) to give a spatially explicit particulates mater-size distribution:

$$\frac{\partial}{\partial t} u(t,x,m) = -div(\omega(x,m) u(t,x,m)). \tag{11}$$

The same initial condition is considered:

$$u(o,x,m) = u^0(x,m), \text{ a.e. } (x,m) \,\varepsilon A = R^3 \times R_4, \tag{12}$$

where $\Lambda = \mathbb{R}^3 \times \mathbb{R}_+$ is endowed with the Lebesgue measure $du = d\mu_{m,x} = dmdx$.

With the assumption:

(**H**): ω is divergence free and globally Lipschitz continuous,

The following conservativeness result is stated:

Result 2

The particulates transport model (11)- (12) is conservative.

Proof: The researcher aims to show that the law (6) is satisfied again. It is proven (DiPerna et al. 1989; Doungmo Goufo 2013; Flandoli et al. 2010) [that under the assumption (H) above, the operator *D*, defined by $D[u(t,x,m)] := -div(\omega(x,m)p(t,x,))$, generates a strongly continuous stochastic semigroup. This yields

$$0 = \int_\Lambda \mathcal{D}u\, d\mu, \quad for\ all\ u \in D(\mathcal{D}),\ then \tag{13}$$

$$0 = \int_{\mathbb{R}^3} \int_0^\infty m^r \mathcal{D}p(t,x,m)\, dmdx, \quad for\ all\ t \geq 0,\ \ r \geq 1.$$

$$\int_{\mathbb{R}^3} \int_0^\infty m\mathcal{D}u(t,x,m)\, dmdx = 0, \quad for\ all\ t \geq 0$$

Thus, which leads to

$$\begin{aligned} \frac{d}{dt}\mathcal{N}(t) &= \frac{d}{dt}\left(\int_{\mathbb{R}^3} \int_0^\infty mu(t,x,m)\, dmdx \right) \\ &= \int_{\mathbb{R}^3} \int_0^\infty m\, \partial_t u(t,x,m)\, dmdx \\ &= \int_{OR3} \int_0^\alpha mDu(t,x,m)\ dmdx \end{aligned}$$

and therefore proving the conservativeness of the transport model.

FINDINGS AND DISCUSSION

When smoke is released by an active smoker in a house room or in a public area, hundreds of billions of airborne solid and liquid particulates and gases are transported and spread in the atmosphere of the room. Soon after that, the smoke becomes invisible and vanishes gradually due natural mechanisms, like air dilution, diffusion and sedimentation. Hence, people sometime have the visual illusion that the toxic chemicals have vanished and the threat to the health is gone. However, the results obtained above tell the researcher that there is always conservativeness of the total number of smoke particulates when the only processes involved are transport and break-up. The ecological interpretation is that these processes do not remove tobacco smoke and its toxic chemicals from the environment where it was emitted. They only change the distribution of particulates in the system. This reinforces the concerns of people are about the problem of passive smoking and the threat it represents for the health. Indeed, about 1.1 billion people around the world are active cigarette smokers, most of them are mothers, fathers, uncles, brothers and other relatives and friends. Thus, the remaining six billion people are either passive smokers or non-smokers at all. This number is huge and the issue of second-hand tobacco smoke represents a great danger for people's health and their environment. Recall that when smoke is release in a room, the visibility becomes obstructed by the cloud of smoke for a few minutes before gradually vanishing, giving the wrong impression that it is gone and the toxicity has ended. This is really a false illusion as mathematically shown above. In the recent years, multiple studies conducted on tobacco smoke exposition confirm that exposure to second-hand tobacco smoke causes illness, disability and death from a wide range of diseases, including various type of cancers. Most of the non-smokers including babies, young kids, teenagers and adults are sometime forced to inhale cigarette smoke despite their will. These victims of second-hand tobacco smoke also face increased risk of related disease and premature death. Human resources are not the only ones affected by the phenomenon. There are also economic burdens on individuals and countries, both for the costs of direct health care as well as indirect costs from reduced productivity. Hence, there is no risk-free level of exposure to tobacco smoke and victims of second-hand tobacco smoke are more likely to die than active smokers.

CONCLUSION

This paper analyses two distinct mathematical systems modeling cigarette smoke dynamics, namely, transport and break-up of atmospheric smoke particulates processes. The researcher has proven that tobacco smoke and its toxic chemicals remain in a house room or place where it has been released and that transport and break-up of atmospheric particulates mater do not remove them. They only change the distribution of the particulates in the room's atmosphere. This study extends the preceding ones with the inclusion of two distinct mathematical models and the results obtained here correspond to the expecting ones.

RECOMMENDATIONS

As said above, there is no risk-free level of exposure to tobacco smoke and the health risk resulting from passive smoking process should be a sufficient reason to ban smoking in workplaces and public places. Parents and relatives also need to be sensitized and educated about the scourge, so as to be aware of the danger incurred by their family. Fortunately, many actions and recommendations in that regard have been undertaken in many countries in the world, especially in developed countries in order to reduce passive smoke exposure, strengthen tobacco control and save lives. However, developing countries, due to lack of infrastructures and sometime lack of willingness, have not yet implemented the WHO policy, namely the World Health Organization framework Convention on Tobacco Control. Accordingly, an increasing number of people living in those countries are still dying due to exposure to tobacco smoke and the researcher hopes to see more legislation that mandates completely smoke-free environments in a very near future.

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