

Error Analysis: Case of Pre-service Teachers

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ABSTRACT The main aim of the study was (1) to identify errors committed by pre-service teachers in Mathematical literacy and (2) to understand why pre-service teachers continue to make such errors. Having used Newman's Error Analysis as the theoretical framework for the study, a four-point Likert scale and a content-based structure-interview questionnaire was developed to address the aforesaid research objectives. The study was conducted by means of a case study guided by the positivists' paradigm with research sample of 105 pre-service university teachers as respondents. In order to test the reliability and consistency of the questionnaires for this study, Cronbach's Alpha was tested for the reliability of standardised items with $\alpha = 0.705$, suggesting an above average reliable instrument. There was moderately weak correlation of $r = +.40$ between reviewing homework before submission and correct answers ($p < .01$). Even though there was a moderate correlation pre-service teachers could not unanimously agree on the confidence when submitting their work. The majority (56%) were indecisive. There was no distinction between those who always took time to answer test questions (42.9%) and those who sometimes (44.8%) did.

INTRODUCTION

In 2006, South Africa implemented a new curriculum; the National Curriculum Statements (NCS) with compulsory mathematics learning in the Further Education and Training (FET) band. All university pre-service teachers are required to study some level of mathematics in either at the foundation or intermediate phase of their teacher education training as one of the fundamental subjects. This is supported by a statement of the Department of Education that:

Mathematical Literacy provides pre-service teachers with awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications in mathematics. It enables pre-service teachers to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems (Department of Education 2003a: 9).

Additionally it is suggested that "*assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of pre-service teachers, using*

various forms of assessment" (Department of Education 2010: 101). It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the pre-service teachers' development in order to improve the process of learning and teaching.

The literature reviewed interrogates "learner errors in Mathematics"; "type of errors for word problems"; the Radatz classification of errors which forms the foundation of the research questions of this study. Examples of learner errors on financial mathematics as cited on the chief marker's report for 2012 also form part of this chapter, as the researcher brings into perspective common learner errors in Mathematical Literacy (in a South African perspective).

As a way forward, this study is aimed at gaining an understanding of pre-service teachers' errors, identifying the underlying factor structure of these errors which will be valuable to the educators' understanding of the identified errors in order to assist in eliminating them at earlier stages of the FET-phase. It will also give a clear outline of the focus of the study; which is Error Analysis of pre-service teachers: a case of financial mathematics by means of the review of the different literatures on error analysis. The types of learner errors in Mathematics would be appropriate to form the focal essence of this

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study, and in conclusion the writer will elaborate on what the study brings to Mathematics Education.

According to the Grade 12 Diagnostic report 2014, the number of candidates writing Mathematical Literacy (ML) decrease by 12 043; candidate passing at the 30 percent level declined by 3.0 percent point whilst candidates passing at 40 percent level declined by 2.9 percent point. Candidates achieving distinctions over 80 percent improved from 1.8 percent to 2.4 percent of the total candidates. This could be attributed to a number of factors such as: pre-service teacher training, error committed by teachers and the introduction of more demanding content and introduction of contexts based on real life situation in Curriculum and Assessment Programme Statements (CAPS).

The inclusion of the pre-service teacher errors is informed by the observation that, pre-service teachers who seem to follow the trend of the lesson plans commit errors when working out the tasks assigned. That stimulated the researchers to critique, understand and find answers as to what type of errors are common in the case of the pedagogy of mathematical literacy. Mathematical literacy accounts for 35 percent weighting of the topics in the examination which indicates that it is more valuable in ML studies. It encompasses a number of basic mathematical skills such as: interpreting, communicating answers and calculating, number and calculations with numbers. This is where pre-service teachers lose marks in their assessment tasks. Mathematical literacy is categorised as the Application Topic which according to DoE (2011: 13) “contain[s] the contexts related to scenarios involving daily life, workplace and business environment, and wider social, national and global issues that pre-service teachers are expected to make sense of content and context”. It includes: financial documents, tariff systems, income, expenditure, profit/loss, income-and-expenditure statements, budget, interest, banking, loans and investments.

For the definitions of errors based on context the researchers sought to relate errors focused on income, expenditure, profit/loss and interest as these are the most common errors committed.

Assessment and Errors

Error is (1) a mistake, (2) the condition of being wrong in opinion or conduct, (3) the

amount of inaccuracy in a calculation or measuring device (Pollard 1994: 270).

Cambourne (1988) as cited in Killen (2007: 3) defined learning as “a process that involves making connections, identifying patterns, and organizing previously unrelated bits of knowledge, behaviour and action into new patterned whole”.

Identifying error is an abstract science of numbers, quantity and space, if pre-service teachers are made aware of the kind of errors they commit and taught skills to avoid them, would that bring about change in their performance in the subject? If these errors keep appearing in all assessments, how much effect do they have on learner performance in formal assessment?

Wiens (2007:1) states that “assessment is such an important piece of educating my students and the careless errors made on these assessments need to be addressed”. Pre-service teachers need to put more focus on and concentration in Mathematical literacy in order to avoid unnecessary loss of marks. “... Teachers’ understanding of learner errors and misconceptions are a key to reform the visions in many countries” (Brodie 2005: 2-178).

Case in Point

The following questions were included in Grade 10 ML pre-service teachers’ assessment task in Simple and Compound interest:

1. How long will it take R5100, invested at 9% simple interest per year to amount to R7 854?

In working out the aforesaid question, most pre-service teachers only calculated 9 percent of R5100 and did not know what to do with that value. Some knew that they have to find the difference between R7854 and R5100, and could not proceed. Brodie (2005: 177) categorised that type of error as missing information.

2. If R12000 is invested at 9.5% simple interest per year, calculate the value of investment after 9 years and three months.

Most pre-service teachers did not have a problem in calculating the interest but they could not calculate it over the stated period. They committed an error in writing 9 years 3 months as

either a common fraction or a decimal. Most of them just wrote 9.3 as the value of period (n). That is an error due to deficient mastery of prerequisite skills, facts and concepts (Radatz 1979: 164).

Problem Statement

Pre-service teachers continue to commit similar errors in their work when they are assessed even with the best teaching and learning strategies. Pre-service teachers are also sometimes not even aware of the errors they commit. There is also a repetitive error in most of their assessment tasks throughout the year. However there are a negligible number of South African studies on the technical know-how of error analysis amongst pre-service teachers.

Specific Objective

To be able to reduce and/or eliminate learner errors, both pre-service teachers and educators in general need to be able to (1) identify them and (2) understand why pre-service teachers continue to make them and then be able to avoid them. The research focused on the mechanisms involved in errors in problem solving.

Literature Review

Understanding the Learning Process

Understanding learning process of can be categorized in two, namely: (1) Instrumental understanding; is demonstrated by someone who uses rules without understanding (rules such as to divide by a fraction you turn it upside down and multiply) (2) Relational understanding occurs when one has built up a conceptual structure of mathematics. "Working memory is especially critical to mathematics learning because mathematics learning places frequent demands on working memory" (Cathercole et al. as cited in Soendergaard and Cachaper 2008:15).

However, the difficulty is that students do not remember intermediate products of calculations in order to solve problems (Eibrink 2008:15). Interconnected problems are more common especially in the income, expenditure and taxation sections. There is also no sufficient good working memory that has been shown to correlate with successful learning.

On the other hand, relational understanding/ thinking occurs when one has built a conceptual structure (Schema) of mathematics and therefore both know what to do and why when one solves a mathematical problem (Eibrink 2008). When dealing with simple and compound interest, the interest may be compounded monthly for three years; that then demands rational thinking of the fact that: three years is thirty six months in trying to find the value of n .

However, there exists a debate among scholars that rational thinking needs to be developed through teaching and learning in the classroom and thus will play a major role in eliminating or reducing the errors committed by pre-service teachers (Peng 2009). There is also unsettling idea that rational thinking needs to be developed through teaching; however, evidence does not support this belief.

The pedagogy of say mathematics does not differ fundamentally across cultures, countries, or gender. The difference is not in the brain or genes, which are basically the same, but in the accumulated knowledge made possible by art, literature and technology. In elaborating on the afore-stated concepts on the learning process of mathematics, this study was based on the two theoretical frameworks expanded below.

The Error Analysis in the Education of Mathematics

Error analysis is more diagnostic, it also allows determining pre-service teachers' lack of basic conceptual understanding. Peng (2009: 1) contends that, "*mathematical errors are a common phenomenon in pre-service teachers' learning of mathematics. Pre-service teachers of any age irrespective of the performance in mathematics have experienced getting mathematics wrong*". It is essential to consider that analysing pre-service teachers' mathematical errors is a fundamental aspect of teaching for mathematics teachers as it will allow them to develop corrective and preventive measures. Based on the aforementioned discussion the concept of error analysis was used in the content analysis of questionnaires.

"Many teachers complain that pre-service teachers find word problems in mathematics more difficult than straight computation and that many pre-service teachers dislike and even fear word problems" (Eibrink 2008: 55). Mathemati-

cal Literacy as described earlier is contextual based, therefore consists mostly of word problems in its nature which explains the existence of persisting learner errors therein. Ryan and McCrae as cited in Radatz (1979: 170) state that: pre-service teachers who confront own mathematical errors, misconceptions and strategies in order to recognize their subject matter knowledge, have an opportunity to develop rich context knowledge.

Radatz (1979: 170) argues that errors in learning are not simply the absence of correct answers or the result of unfortunate accidents. The author suggests that they are the consequences of definite processes whose nature must be discovered. What is however impossible to analyze is the nature and underlying causes of errors in terms of the individuals' information-processing mechanisms. Also difficult is the analysis of errors which offers a variety of points of departure for research into processes by which children learn mathematics.

What is nonetheless clear is that errors can be the result of carelessness; misinterpretation of symbols or text; lack of relevant experience or knowledge related to the mathematical literacy topic/concepts; a lack of awareness or inability to check the answers given; or the result of a misconception.

Another challenging view is that, pre-service teachers who have difficulty learning typically lack important conceptual knowledge for a number of reasons, including an inability to process information at the rate of the instructional pace; lack of adequate opportunities to respond; the lack of specific feedback from the teacher regarding the misunderstanding.

Thus by pinpointing learner errors teacher can provide instruction targeted to the pre-service teachers' area of need.

Theoretical Framework

The researchers were guided by two theoretical frameworks: Polya's problem-solving techniques and Newman's error analysis in deconstructing the concept of error analysis.

Polya's Problem-solving Techniques

Polya developed four basic principles that need to be considered during problem solving. Based on the principles, the four steps that need

to be followed during problem solving were developed later. The current study seeks to identify a problem which seeks to solve a particular problem; namely that pre-service teachers continue to commit errors in a particular case such as Mathematical literacy. This might be attributed to the pre-service teachers' problem-solving techniques; understanding of the aforementioned theory can assist in eliminating the errors. Polya's problem-solving techniques are a theory that describes four steps to be followed during problem-solving in mathematics education.

According to Polya the following are the principles to be considered during problem solving:

First principle: Understand the problem

Second principle: Devise a plan

Third principle: Carry out the plan

Fourth principle: Look back: Taking time to reflect on your work enables you to predict the relevant strategy for solving a future problem.

If the teaching and learning process at schools could be influenced by the above-stated theory, both teacher and pre-service teachers guided by the four-step principle, could dedicate more time to pre-service teachers' work and thus reduce errors committed.

Newman's Error Analysis

The researchers were guided by Newman's Error Analysis technique. Newman's Error Analysis (NEA) provided a framework for considering the reasons that underpin the difficulties students experience with for instance word problems and a process that assist teachers to determine where misunderstandings occurs. NEA also provides directions where teachers could target effective teaching strategies to overcome them (White 2010: 129-148). In the search for underlying factors for pre-service teachers committing the errors and finding the suitable strategies to overcome them, the researchers will be guided by NEA. The Newman's error analysis and follow-up strategies have helped pre-service teachers with their problem-solving skills, and teachers developed a much more consistent approach to the teaching of problem solving. "Not only has it raised awareness of the language demands of problem solving, but through its systematic approach, teachers can focus on teaching for deeper understanding" (White 2009: 37).

METHODOLOGY

Both interpretive and positivist paradigm were used for the measurement of data in order to discover and confirm causes and effects. The research was conducted with pre-service teachers across Eastern Cape Province in South Africa. The random sample of 105 pre-service teachers teaching in various schools was considered. Data was collected by means of a structured-interview questionnaire and documentary studies (examiners reports). The documentary analysis was based on why do pre-service teachers commit errors on given tasks in Mathematical literacy. Additionally, a structured-interview instrument was used; and it included content based questions to uncover the possible underlying factors related to the different types of errors pre-service teachers commit. The second questionnaire contained the rating scale questions using a Likert scale were used to collect data from the respondents. The researchers sought to analyze the trends in pre-service teachers' thinking and related known characteristics to commit errors. Quantitative analysis with descriptive statistics which describe the distribution, the relationship among variables and variability through the use of frequencies was used to analyse the fourth questionnaire. Statistical Package of Social Sciences (SPSS) version 21 was used for correlation coefficient analysis to measure the relationship between variables of each of the aforesaid research questions.

Reliability and Validity of the Study

In order to test the reliability and consistency of the questionnaires for this study, Cronbach's Alpha was tested for the reliability of standardised items with $\alpha = 0.705$, suggesting an above average reliable instrument. For validity, the researchers ensured that the variables were isolated and controlled in the sampling stage; the sample was randomly selected to avoid any influence on the study.

Ethical Considerations of the Study

Permission was requested from the university and the various schools in which the pre-service teachers taught. The permission was granted with no conditions attached. Consent from participants was also sought. Informed

consent forms included a brief outline of the nature of the study, a description of what would be the participants' involvement, the duration of the study, the researcher's name and contact details, signature and date of the letter of agreement. The data collected was treated with confidentiality and protected. The data was kept under strict care of the researchers. The rights of the participants were protected through anonymity, confidentiality and privacy.

FINDINGS

Based on the analysis of the frequencies of the variables in afore-stated research question it was revealed that the majority (76.2%) of the respondents always wanted to do their best in a test but from the sample only 35.2 percent of the respondents always felt confident when submitting their test scripts, whereas 56.2 percent sometimes felt confident. Only 8.6 percent attested that they always felt rushed when writing a test while 47.5 percent sometimes felt rushed but about 28.6 percent of the respondents sometimes wanted to be the first ones to finish writing, whereas 10.5 percent always wanted to be the first to finish writing. The majority which is 83.1 percent of the respondents claimed to go back and read through what they had written even though the study revealed that only 47.6 percent always did that. From the summary of the responses it was ascertained that 87.7 percent of the respondents took time to answer test questions. The majority which is 81 percent of the respondents reviewed their homework but only 36.2 percent always did that, which means 45.5 percent occasionally did it.

This study specifically sought the reason behind pre-service teachers committing the kind of errors they do when dealing with financial mathematical problems in Mathematical Literacy. A number of variables were tested for this research question, and they ranged from interest in doing their best in a test, confidence when writing tests, wanting to be the first to finish writing, not going back to read through what was written. In describing the errors Radatz (1979: 170) states that: "Errors in learning of mathematics are not simply the absence of correct answers or results of unfortunate accidents. They are the sequence of definite processes whose nature must be discovered".

It was evident in the correlation results which showed a moderate correlation between the two variables. Even though there was a moderate correlation between the two variables, pre-service teachers could not unanimously agree on the confidence when submitting their work. The majority (56%) were indecisive and stated that they did not always feel confident. The fact that they did not feel confident was because they felt rushed when writing a test. A minority (10.5%) sometimes wanted to be the first ones to finish writing. Pre-service teachers lacked confidence when submitting the tests and that was illustrated in the results of this specific variable. It is attributed to the following: There was no distinction between those who always took time to answer test questions (42.9%) and those who sometimes (44.8%) took time when answering test questions. Taking time to answer questions showed a weak correlation with going back and read through what they had written, which indicated that according to the correlation results there was no relationship. Even though they took their time to answer a question that does not mean they spent some time reading what they had written. Among 105 respondents only 47.6 percent attested that they always went back and read what they had written before handing in their scripts. It also illustrated a negative weak correlation to the other variables when tested which indicated that it had no relationship with those variables. Only 42.9 percent of the respondents attested to always taking time to answer the test questions, which left 57.1 percent who never or sometimes did so.

The participants had divided opinions on whether they reviewed their homework as 45.7 percent did not always review their homework whereas 36.2 percent always reviewed their homework. The uncertainty of the pre-service teachers' responses was evident in the correlation results which all indicated weak or no correlation between the variables. Reviewing homework before submission was statistically significant with $p < .01$ with a moderate correlation of $r = +.40$.

DISCUSSION

Based on the analysis, pre-service teachers felt rushed when writing a test and therefore they panicked and rushed to finish writing without spending enough time in answering test

questions. Statistical tests and recorded frequencies revealed that some pre-service teachers felt rushed when writing a test even though the results could not significantly support that, but based on frequencies it is worth considering, 47.6 percent of the sample considered that as an underlying factor related to committing errors. Sometimes they did not go back to read through what they had been written, and that was apparent from the careless mistakes obvious in their work (White 2008). There was no relationship between going back to read through the work and wanting to do their best in a test (Wiens 2007; Peng 2009). The majority did not always review their homework after completing it (Radatz 1979; DoE 2011). This could be attributed to the time allocated to do the homework and conditions under which the pre-service teachers did their homework. Therefore, to avoid committing errors pre-service teachers could be taught the four steps to follow when given a task.

Learners should be discouraged from providing responses learnt by rote without judiciously assessing whether the response is relevant as this could be related to the errors committed (DoE 2014).

Pre-service teachers should be taught to apply Polya's problem-solving techniques. That will train them in applying the techniques to make sure they understand the question before attempting to answer it; to plan before answering; to answer and then review what was written to make sure that they reduce the errors committed.

CONCLUSION

Although respondents wanted to do their best in assessment tasks, there was a weak relationship with reviewing their homework.

- ♦ The study revealed that respondents sometimes felt rushed, therefore panicked, when they wrote tests and made mistakes.
- ♦ Respondents felt confident when submitting their test script, which was also related to them finishing first and as a result committing errors in their work.

RECOMMENDATIONS

From the aforementioned, error analysis may be incorporated in the teacher training curricu-

lum as it will assist in reducing or eliminating learner errors. It will assist educators to be able to identify learner errors, assist pre-service teachers in eliminating those errors.

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