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An Assessment of the Readiness of Grade 10 Learners for Geometry in the Context of Curriculum and Assessment Policy Statement (CAPS) Expectation

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ABSTRACT In 2012 January, as part of the ongoing process of curriculum revision which began in 1994, the South African Depart of Education (DoE) introduced the Curriculum and Assessment Policy Statement (CAPS). The CAPS brought Euclidean geometry with its formal proof back into the compulsory mathematics curriculum. It also implied that all learners needed to perform at level 4 (Deduction: with formal proof) of the van Hiele levels in Euclidean geometry in all grades in the Further Education and Training (FET) phase-Grades 10-12. The main aspect of the van Hiele theory is that of children's understanding of geometric concepts can be characterised as being at certain specific levels within a hierarchy of levels from 0 to 4 with level 0 being the lowest. This paper reports on the assessment based on the van Hiele model of geometrical thinking levels of a sample of South African Grade 10 learners in Euclidean Geometry. It is a follow up of an earlier published study with 191 grade 10 learners by the same authors. Data for this study were generated from 359 Grade 10 learners in five senior secondary schools in one Education District. These schools were selected through purposive sampling. The study made use of both quantitative and qualitative research techniques for data collection. In the former, multiple choice questions and in the latter, structured interviews were made use of. The quantitative and qualitative data were anlysed using Microsoft Excel 2007 and thematic analysis, respectively. The results from the study revealed that the majority of the learners were at level 0 despite the CAPS expectation that learners are to perform at level 3 in order to be ready for level 4 thinking in Grade 10. The findings from the study assisted the authors to suggest recommendations that could be made use of by curriculum developers and implementers to improve the instructional strategies of geometry learning and teaching.

INTRODUCTION

The South African Constitution (1996) laid the basis for social transformation in post-apartheid South Africa. To attain social transformation, the South African Government attached a great deal of importance to the learning and teaching of mathematics, science and technology (MST) in the South African schools. According to the South African Department of Education (DoE 2003), it was deemed vital that all learners passing through the Further Education and Training (FET) phase (Grades 10 - 12) acquire a functioning knowledge of mathematics that empowers them to make sense of the society.

Recent research outputs show continued interest in mathematics education in general and geometry education in particular (see for example, Patsiomitou et al. 2010; Hulme 2012; Abdullah and Zakaria 2013; Naidoo 2013; Özçakir 2013; Patsiomitou 2014). Abdullah and Zakaria opine that "Mathematics is also a discipline that trains the students to think logically and systematically in solving problems and making decisions" (Abdullah and Zakaria 2013:4433). School mathematics being a gateway subject to several tertiary studies, adequate learning facilitation in this subject is of pivotal importance in any country (van der Walt and Maree 2007). Mathematics ensures access to an extended study of the mathematical sciences and a variety of career paths (DoE 2003) and geometry is an essential part of the mathematics curriculum.

Research into learning and teaching of geometry and geometrical proof is ongoing (Patsiomitou et al. 2010; Abu and Abidin 2013; Naidoo 2013; Özçakir 2013; Meng and Idris 2012; Meng and Sam 2013; Bal 2014). Patsiomitou et al. (2010) described a two- fold teaching experiment with respect to geometric proof in grade 10 in Greece. Within South Africa, Naidoo (2013) explored the influence of social class on learners' reasoning

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in geometry at two schools in South Africa's KwaZulu-Natal province. Hulme (2012) researched on the role of technology in the zone of proximal development and the use of Van Hiele levels as a tool of analysis in Grade 9.

Geometric skills are important in architectural design, physics, astronomy, art, geology, mechanical drawing, engineering, and in various areas of construction work. The fields of study mentioned above play a major role in the development of any given country. It is for the above mentioned reasons it was envisaged that the South African learners should study geometry as part of their experience with mathematics in order for them to have a wide range of options in choosing appropriate higher education courses, career paths and occupations. Despite geometry being an important branch of mathematics, there are many challenges in learning and teaching it.

Ongoing Educational Policy Revisions, Modifications and Reformations

Since the inception of the new democratic government in South Africa in 1994, the DoE embarked upon a number of educational policy revisions, modifications and reformations. This process resulted in the implementation of an interim core syllabus in 1995 which was succeeded by Curriculum 2005 (C2005) in 1998 (King 2003). This was followed by a curriculum review in 2000, which culminated in the release of a document called "Draft National Curriculum Statement" (NCS) by the Minister of Education in 2001 and later, a Revised National Curriculum Statement (RNCS) in 2002 (King 2003). RNCS came in to effect in the Further Education and Training Band (FET) in 2006, where Euclidean Geometry was excluded from the compulsory mathematics curriculum component.

The NCS emphasised learning outcomes (DoE 2003). Each subject had its own learning outcomes and each learning outcome had its own assessment standards. A learning outcome described the knowledge, skills and values the learner had to acquire in a phase and assessment standards were considered as criteria that defined the knowledge, attitude, values and skills that a learner had to know and be able to demonstrate at a specific grade. There were five learning outcomes in mathematics, namely, Learning Outcome (LO) 1, LO 2, LO 3, LO 4 and LO 5. One of the mathematical learning outcomes (LO 3) was the mastery in space and shape (DoE 2003). Within the NCS, geometry was part of the attainment target currently entitled as 'space, shape and measurement'. A description of Learning Outcome 3 stated, "... the treatment of formal Euclidean geometry is staged through the grades so as to assist the gradual development of proof skills and an understanding of logical axiomatic systems" (DoE 2003:54). The above criteria of LO 3 was closely linked to levels 1, 2, and 3 and to a certain extent of level 4 (means no formal proof is required for examination purposes) of van Hiele levels of geometric thinking (see later). The above prescribed learning outcome 3 was part of the compulsory paper 2 in the National Senior Certificate examination in mathematics. In addition to this, learners in Grade 12 could opt for an additional optional paper (paper 3), which examined optional assessment standards in LO 1, LO 3, and LO 4. The optional assessment standards in LO 3, contributed 40% of the examination mark which comprised of learners learning different aspects of Euclidean geometry including proving theorems (formal proof) in similarity, proportionality and circle geometry they would have learned in grades 11 and 12.

A further revision of the curriculum called Curriculum and Assessment Policy Statement (CAPS) came into effect in the FET phase in Grade 10 in 2012. CAPS replaced the old Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines. The Department of Basic Education (DBE) as the new government department dealing with school education clarified that the amendment to the NCS was to improve implementation (DBE 2011). Optional assessment standards such as formal proof in Euclidean Geometry were brought back into the compulsory curriculum of paper 2. The CAPS document (DBE 2011) states that learners are to "investigate line segments joining the midpoints of two sides of a triangle and properties of special quadrilaterals" (DBE 2011:14). Learners are expected to define, investigate and make conjectures and prove conjectures on special quadrilaterals (DBE 2011:25). Thus the CAPS document implied that all learners are to perform fully at a higher level (level 4 – Deduction: with formal proof) of the van Hiele levels in major aspects of Euclidean Geometry in all grades in the FET phase.

Researchers have taken interest in exploring different aspects of CAPS mathematics curriculum implementation. For example, Lumadi (2013) began discussion on the power which teachers have in mathematics curriculum assessment in the FET level. This paper reports on a study which made use of the van Hiele levels of geometric thinking to assess the readiness of grade 10 learners for Euclidean geometry. It is a follow up of an earlier published study with 191 grade 10 learners by the same authors (Alex and Mammen 2012). The main research question was 'Are the grade 10 learners ready for geometry in the context of CAPS?'

The van Hiele Theory

Experience in classroom teaching in the Netherlands in the 1950s, influenced the husband and wife van Hiele team (Pierre van Hiele and Dina van Hiele – Geldof) to put forward a theoretical perspective for the teaching and learning of geometry. This theory is universally referred to as the van Hiele theory (Pegg and Davey 1998). The van Hiele theory was primarily directed at improving teaching as well as the geometric understanding of learners by organising instruction in such a way that it would take learners' thinking ability into account whilst the new work is being introduced. This theory is particularly relevant in South Africa, where mathematics remains a problematic learning area, as Fuys et al. (1988:191) suggest that "its emphasis on developing successively higher thought levels appears to signal direction and potential for improving the teaching of mathematics".

The van Hiele levels are:

Level 1-Recognition: Students recognise a figure by its appearance (or shape/form). It is the appearance of the shape that defines it for the student. A square is a square, "because it looks like a square". And a child recognises a rectangle by its form and a rectangle seems different to him than a square (van Hiele 1999:311), or, "It is a rectangle because it looks like a door" (van der Sandt and Nieuwoudt 2005:109). Since the appearance is dominant at this level, appearances can overpower properties of a shape.

***Level 2- Analysis*:** Students at this level are able to consider all shapes within a class rather than a single shape. By focusing on a class of shapes, students are able to think about "what makes a rectangle a rectangle" (van de Walle 2001:309). Students at this level identify a figure by its properties, which are seen as independent of one another (Pegg and Davey 1998). Class inclusion is not yet understood.

***Level 3-Informal Deduction*:** Students at this level discover and formulate generalisations about previously learned properties and rules and develop informal arguments to show those generalisations to be true (Malloy 2002). They no longer see properties of figures as independent. They recognise that a property proceeds or follows from other properties. They also understand relationship between different figures (Pegg and Davey 1998). But the role and importance of formal deduction, however, is not yet understood (Mason 1998).

***Level 4-Deduction*:** Students at this level prove theorems deductively and understand the structure of the geometric system (Fuys et al. 1988; Malloy 2002). The students reason formally within the context of a mathematical system, complete with undefined terms, axioms an underlying logical system, definitions, and theorems (Burger and Shaughnessy 1986). They use the concept of necessary and sufficient conditions and can develop proofs rather than learning by rote. They can devise definitions (Pegg and Davey 1998). They are able to make conjectures and prove them (De Villiers 2003).

Level 5- Rigor: This is the highest level in the van Hiele hierarchy (Teppo 1991). Students at this level can establish theorems in different systems of postulates and compare and analyse deductive systems (Fuys et al. 1988; Malloy 2002).

Two different numbering systems are commonly used in the literature to describe the van Hiele levels: level 0 to level 4 (Atebe 2008) and level 1 to level 5 (Senk 1989; Pegg and Davey 1998; Malloy 2002). In this paper, all references and all results from the research using the 0 – 4 numbering system have been changed to the numbering system as level 1 to level 5. Meanwhile, as a result of learners not achieving even the basic level (level 1), such as 'not yet at level 1' or 'weak level 1', researchers have suggested the introduction of another level, called level 0 (pre-recognition level). Clements and Battista (1992:429) defined it as "children initially perceive geometric shapes, but may attend to only a subset of a shape's visual characteristics and they are unable to identify many common shapes". The present study limits its scope to the first four levels namely, visualisation, analysis, informal deduction and deduction (labelled as level 1-4), with the possibility of the inclusion of level 0 (pre-recognition).

METHODOLOGY

This evaluation study employed mixed methods for data collection-both quantitative and qualitative. The readiness of the grade 10 learners for geometry in the context of the CAPS was established through a test on the van Hiele levels of thinking among 359 grade 10 learners from five purposively selected schools from the senior secondary schools in Mthatha in the Eastern Cape Province of the Republic of South Africa. Geographical accessibility, proximity and functionality were some of the factors that influenced the choice of these schools. The learners were selected using convenience sampling. They were selected on the basis of being accessible. Since all learners from each school in grade 10 were involved in the study, the numbers of the sample per school were unequal. This choice was considered as more appropriate to get a general representative overview of grade 10 learners' van Hiele levels rather than choosing equal numbers from every school. Interviews with selected learners from the five schools on their levels of thinking were also conducted to enrich the study by giving it a qualitative flavour.

Instruments for Quantitative and Qualitative Data Collection, Mode of Data Collection and Analysis

The Quantitative Component: Van Hiele Geometry Test

The van Hiele Geometry Test (VHGT) which was adapted with permission from a similar study (Atebe 2008) done in the Grahamstown area in South Africa, which was adapted from the CDASSG Project of Usiskin (1982) was used to gauge the learners' readiness for geometry. The VHGT test comprised of multiple choice questions (MCQs) in four subtests. Each subtest consisted of 5 items based on one van Hiele level. There were 20 items in the test, with item numbers 1-5, 6-10, 11-15, and 16-20 to test learners' attainment of van Hiele levels 1, 2, 3, and 4 respectively. Sample items are shown in Appendix A. The mathematics educators in the five targeted schools distributed the test question papers and MCQ answer sheets and collected back the question papers and completed answer sheets after the test. Each correct response was assigned 1 mark and hence, highest score could be 20. The scores were captured on Microsoft Excel 2007 and the percentage scores were calculated. The grading of the VHGT was done again using a second method which was based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982: 22). By this criterion, if a learner answered at least 3 out of 5 items in any of the 4 subtests within the VHGT correctly, then he/she was considered as having mastered that level. According to this grading system the learners' scores were weighted as : 1 point for meeting criterion on item 1-5 (level 1); 2 points for meeting criterion on item 6-10 (level 2); 4 point for meeting criterion on item 11 - 15 (level 3); 8 point for meeting criterion on item 16-20 (level 4). This could make the maximum score for any learner to be 1+2+4+8 = 15 points. This weighted sum helped to determine the van Hiele levels at which the criteria were met from the weighted sum alone. For example, a score of 7 would indicate that the learner met the criterion at levels 1, 2 and 3 (that is, 1+2+4 =7). This grading system helped to assign the learners into various van Hiele levels based on their responses. A weighted sum of 0 would indicate that a learner has not achieved any levels, as the learner did not get at least 3 out of any subtests of the VHGT. The learners' performance in the VHGT was taken as the measure to find the geometrical thinking level of the learners. The data were first analysed in terms of the percentage mean and then in terms of the percentage of number of learners in each level of the van Hiele levels according to the criterion developed by Usiskin (1982).

The quantitative instrument that was used in this part of the research was one that was used earlier by Atebe (2008) for a doctoral study. He had used the split-half method to check the reliability. Nonetheless, the validity was further tested by its review by two experts in geometry.

The Qualitative Component: Interviews

The qualitative component consisted of structured interviews of 30 learners from the sample, six from each school. Each interview consisted of giving the learners seven open ended tasks dealing with geometric shapes, developed by Burger and Shaughnessy (1986), which were designed to reflect the descriptions of the van

Hiele levels. These tasks were used by Genz (2006: 57-58) and the authors used these tasks for the study through adoption of the questions and simple adaption by altering the numbering of figures in the activity. The tasks are shown in Appendix B. The tasks involved drawing triangles and quadrilaterals, identifying and defining shapes, sorting shapes and engaging in informal and formal reasoning about geometric shapes. These tasks were expected to draw out the characterisations of van Hiele levels 1 to 3 (Burger and Shaughnessy 1986). The individual interviews lasted approximately 40 to 60 minutes and took place in the learners' classrooms after school hours in order not to disturb the normal school activities. One of the authors conducted all the interviews.

Ethical Compliance

Ethical compliance was achieved through obtaining a formal permissions from the Department of Education and from the principals of the targeted school, distribution of a research information sheet to the members of the sample and parents/guardians, signed voluntary informed consent forms (including, among others, the freedom to withdraw from the study at any stage) from members of the sample and parents of those under 18 years and adherence to anonymity of both the schools and members of the sample.

RESULTS

Quantitative Data

Table 1 showed that the majority of the learners were at level 0 (56%). For the van Hiele levels 1, 2, 3, and 4, the percentages were 26%, 17%, 1% and 0% respectively. As can be seen from Table 1, the majority of the learners in all schools were at level 0 except for school D which had only 29% at level 0. School C had the highest number of learners at level 0 (70%) followed by School E (65%), School B (63%) and School A (54%). Level 3 was not achieved by any learner in any school except 6% at School D. None of the schools had learners at level 4.

In a study by the same authors, during the year preceding the present study, where 191 grade 10 learners from the same schools were the sample, 48% of learners were found to be at level 0. This deficiency was indeed pointed out to the respective school authorities when that study was concluded (Alex and Mammen 2012).

Qualitative Data

The data for the learners' interviews consisted of the learners' drawings, the interviewer's field notes and the audio taped interviews. The highlights from only two members of the sample (one female and one male), employing pseudo names to conform to anonymity, are included in order to comply with limitation of length of the paper. The interviewee responses suggested that many learners were only attending to the visual characteristics of the shapes. Excerpts such as "they look like triangles" and "they look like squares" were common in conversations with learners who were at levels 0 and 1. For example, on the activity to identify and name triangles (Appendix B), the following transcript represents data from Andiswa based on what she marked on the figures:

Researcher: *Andiswa, Why did you put a "T" on No. 5?*

Andiswa: *Because... it is a 'quadrilateral triangle'*

Researcher: *Why do you say so?*

Andiswa: *Because "both sides" are equal.*

Researcher: *Why didn't you put a "T" for No. 3 and No.7?*

Andiswa: *They do not "look like" triangles.*

Table 1: van Hiele level of geometric thinking of learners in the five schools and all schools

	Number of learners and their percentages at each van Hiele level:All schools					
Van Hiele levels	*School A(N and %)*	*School B (N and %)*	*School C (N and %)*	*School D (N and %)*	*School E (N and %)*	*Total (N and %)*
Level 0	42 (54%)	67 (63%)	40 (70%)	19 (29%)	34 (65%)	202 (56%)
Level 1	18 (23%)	29 (27%)	15 (26%)	22 (34%)	7 (14%)	91 (26%)
Level 2	18 (23%)	11 (10%)	2 (4%)	20 (31%)	11 (21%)	62 (17%)
Level 3	0	0	0	4 (6%)	0	4 (1%)
Level 4	0	0	0	0	0	0 (0%)
Total	78 (22%)	107 (30%)	57 (16%)	65 (18%)	52 (14%)	359 (100%)

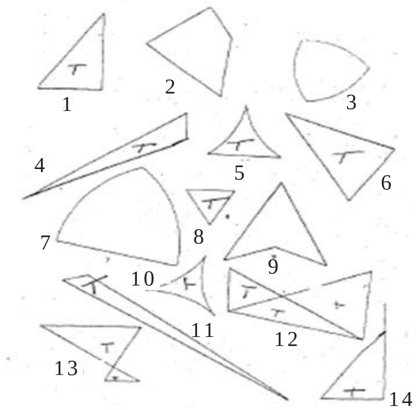

Fig. 1. Sample interview transcript from a female learner

This showed that she did not use the properties when she focused on identifying them (for example, No.5) and could not identify certain triangles. To elicit the properties that the learner perceived as necessary for a figure to be a triangle, the following transcript is evidence:

Researcher: *If you want your little sister to look for a triangle from this paper, what will you tell her to look for?*

Andiswa: *She should look for 'a figure with 3 sides'.*

Researcher: *What if she picks No.3 and No.7 also?*

Andiswa: Oh...ya...(giggling ... , thinking for a while), *she must look for a 'triangle with 3 sides'* (by pointing to the figures she marked in the paper).

The following transcript is from the interview with Mila on the same activity:

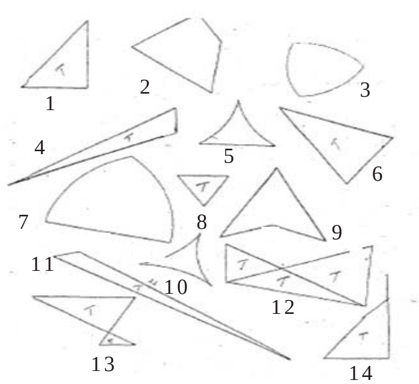

Fig. 2. Sample interview transcript from a male learner

Researcher: *Mila, why did you put a "T" on No.4, and No.6?*

Mila: Mam... *Because they have 'two sides equal' and 'one side not equal'...they have three sides.*

Researcher: *Why didn't you put a "T" on No.5 and No. 10?*

Mila: *They are not triangles.*

Researcher: *Why is No. 3 not a triangle?*

Mila: *Oh... Mam...I think... Mam...It is not a triangle because 'it looks circular shape... but not pointy'.*

Researcher: *How is No.3 different from No.1?*

Mila: *No.1 'has 90^0 angle and sharp corners than No.3'.*

DISCUSSION

Quantitative Data

The analysis of the levels of thinking showed that most of the learners were at level 0. None of the schools had learners at level 4 thinking on the van Hiele level indicating that the learners were not ready for formal geometric proofs in grade 10. Despite the data being those from two studies in successive years, the percentage of learners at all levels were comparable considering that there were 168 learners more in the present study (N=359 vs N=191). Furthermore, despite warning bells being sounded to the schools after the results of the previous study, no noticeable improvements could be traced in the latter study. Most learners continuing to be at level 0 is indeed a matter of concern. The findings were also consistent with earlier studies of Usiskin (1982), Siyepu (2005) and Atebe (2008). More recent studies by Meng and Idris (2012), Abu and Abidin (2013), Meng and Sam (2013) and Bal (2014) also reported similar observations. It was evident from the present study that the majority of the learners was also not reaching the level set by the curriculum, which expected the learners to be operating at level 3 and level 4.

Qualitative Data

The interview transcript of Andiswa, indicates that she had not reached visual, analysis and informal deduction levels of thinking. For her, the properties that she perceived as necessary for a figure to be a triangle were not clear.

Anything that 'looks like a triangle' was a triangle for her. No.5 was a 'quadrilateral triangle' because 'both sides were equal'.

In the case of Mila, although he could identify certain triangles, he did not use the properties when he focused on identifying them, indicating that he had not reached the analysis level and informal deduction level of thinking.

The two sample responses indicate that the learners in grade 10 operate at level 0 or level 1 since they only attended to the visual prototypes to characterise shapes. The use of imprecise properties to compare shapes was very prevalent. This inference is consistent with earlier studies (van Hiele 1986; Burger and Shaughnessy 1986; Pegg and Davey 1998; van de Walle 2001), where the characteristics of level 1 thinking were documented.

The analysis of the responses from the interviews suggested that many learners had difficulty with the ordering of the properties of simple geometric shapes. The data also supported the claim of Mayberry (1983) that high school learners did not perceive the properties of shapes and that of Burger (1985) that many learners relied on imprecise qualities to identify shapes like 'pointy triangles' and 'slanted squares'.

Previous international studies (for example, Usiskin 1982; Mayberry 1983; Burger 1985; Burger and Shaughnessy 1986; Fuys et al. 1988; Renne 2004) and South African Studies (for example, Feza and Webb 2005; King 2003) point out that many learners in the middle school have severe misconceptions concerning some important geometric ideas. South African research reports (for example, De Villiers and Njisane 1987; Atebe 2008) indicate that high school learners in general and more especially, Grade 12 learners are functioning below the levels that are expected of them, that is, they are at concrete and visual levels rather than at abstract level in geometry. This was noted predominantly at level 0 and level 1 thinkers in the interviews. De Villiers and Njisane (1987) pointed out that the transition from concrete to the abstract level of thinking posed "specific problem to second language speakers" and success in geometry involved the acquisition of the technical terminology. The language competency had a negative impact in the attainment of higher levels of understanding in geometry. It is indeed a necessity to establish connections between relationships of mathematical concepts and terminology.

All the aspects that are discussed from the interviews are of importance to instruction as they are of big concern which affects the understanding of mathematics in general and geometry in particular. It also appeared that the learners from different schools involved in the study had varied exposure to geometric figures and their characteristics.

CONCLUSION

The curriculum reforms and changes that have been implemented in South Africa within the past 10 years and the latest addition, CAPS have added major changes to the curriculum. CAPS implies that learners are to be at level 3 but the evidence of only 1% learners at level 3 and 0% at level 4 is indeed a cause for concern. As such in grade 10, learners are not ready for formal proof in Euclidean geometry as they are expected to be operating at these levels, that is, they are not sufficiently grounded in basic geometric concepts and relations. Language incompetency in general and inadequate technical mathematical vocabulary are barriers to the attainment of higher levels of understanding in geometry. The findings of the study lead to the importance on the delivery of instruction that is appropriate to learners' level of thinking. Junior secondary school geometry curriculum implementers are not adequately preparing the learners to face the challenges in the senior secondary school.

RECOMMENDATIONS

Junior secondary school geometry teaching should enable the learners to develop visual skills related to common two and three dimensional figures. Learners should be engaged in the activity of defining and be allowed to choose their own definitions and educators then need to lead them to the correct definitions with understanding, that is, leading learners from a context-embedded environment into the context-reduced environment in the abstract. It is necessary to design appropriate experiences for pre-service and in-service educators to familiarise themselves with the van Hiele theory to enable them to design and use appropriate material for instruction according to the levels of their learners. Educators need to be conversant with the van Hiele theory in order to assist learners in formulating appropriate geometrical activities for their learners. If educators are successful in introducing a van Hiele theory based instruction, learners can be prepared for their challenges in the CAPS.

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APPENDIX A

Sample questions from van Hiele level 1 subtest:
Question1. Which of these are triangles?

Fig. 1. Sample item from van Hiele level 1 subtest

A. All are triangles
B. 4 only
C. 1 and 2 only
D. 3 only
E. 1 and 4 only

Sample question from van Hiele level 2 subtest:
Question 10: RSTU is a square. Which of these properties is **not true** in all squares?

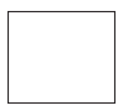


Fig. 2. Sample item from van Hiele level 2 subtest

A. RS and SU have the same measure.
B. The diagonals bisect the angles.
C. RT and SU have the same measure.
D. RT and Su are lines of symmetry.
E. The diagonals intersect at right angles.

Sample question from van Hiele level 3 subtest:

Question 12: Which is **true**?
A. All properties of rectangles are properties of all parallelograms.
B. All properties of squares are properties of all rectangles.
C. All properties of squares are properties of all parallelograms.
D. All properties of rectangles are properties of all squares.
E. None of (A) – (D) is true

Sample question from van Hiele level 4 subtest:
Question 17: Examine these statements.
i). Two lines perpendicular to the same line are parallel.
ii). A line perpendicular to one of two parallel lines is perpendicular to the other.
iii). If two lines are equidistant, then they are parallel.
it is given that lines S and P are perpendicular and lines T and P are perpendicular.

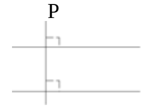


Fig. 3. Sample item from van Hiele level 4 subtest

Which of the above statements could be the reason that line S is parallel to line T?
A. (i) only
B. (ii) only
C. (iii) only
D. Either (ii) or (iii)
E. Either (i) or (ii)

APPENDIX B

Adopted and adapted from Genz (2006: 57-58)

Identifying and Defining Triangles (Genz, 2006).

Part A

Purpose: To determine whether the student can identify certain triangles.
Script: Put a T on each triangle on this sheet

Part B

Purpose: To determine the properties that the student focuses on when identifying triangles.
Script:

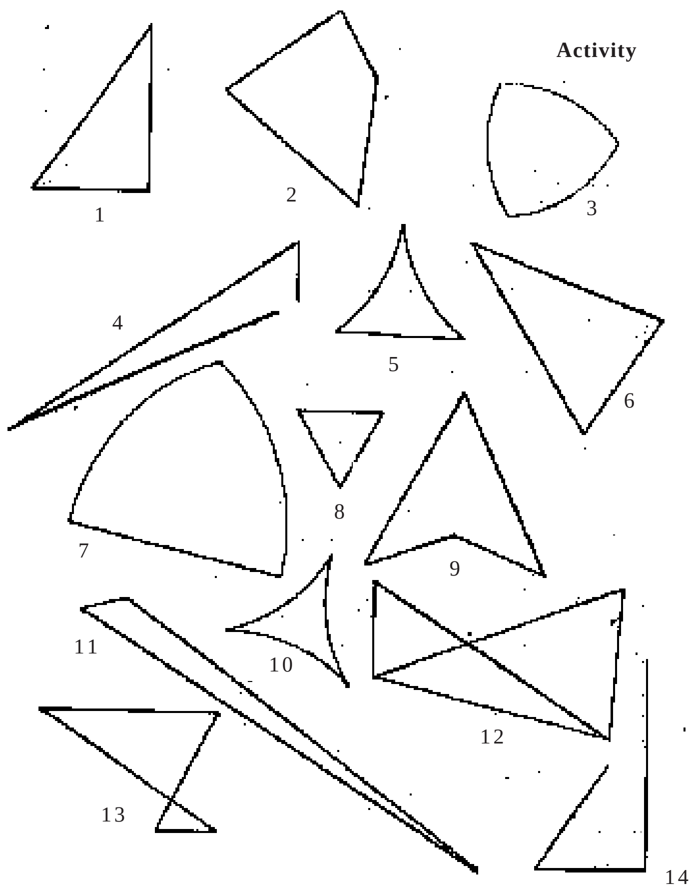
1. Why did you put a T on _____________?
   (Pick out at least ¾ of those marked) Be sure to include all "unusual" responses.
2. Are there any triangles in #12? If so, how many do you see?
3. Are there any triangles in #13? If so, how many do you see?
4. Pick out at least 4 (if possible) not marked as triangles. Ask, why you did not put a T on _________ _________ ?(for each one)

Part C

Purpose: To elicit properties the student perceives as necessary for a figure to be a triangle
Script: What would you tell someone to look for to pick out all the triangles on a sheet of figures?

Part D

Purpose: To elicit properties the student perceives as necessary and sufficient for a figure to be a triangle.
Script: What is the shortest list of things you could tell someone to look for to pick out all the triangles on a sheet of figures?



The above figure is from Genz (2006:58).
(Genz RL 2006. *Determining High School Geometry Students' Geometric Understanding Using van Hiele Levels: Is There a Difference Between Standards-based Curriculum Students and NonStandards-based Curriculum Students?* MA. Dissertation. Provo, Utah: Brigham Young University)